4. Nonparametric estimation by kernel methods

Task 1. */1 point/* Generate $3N$ random samples in every case:

1) N samples from $\mathcal{N}(0,1)$, N samples from $\mathcal{N}(5,0.25)$, N samples from $\mathcal{N}(15,1)$,
2) N samples from $\mathcal{U}[0,2]$, N samples from $\mathcal{U}[1,1.5]$, N samples from $\mathcal{U}[3,4]$,
3) $3N$ samples from beta distribution, eg.

```
x = [4+betarnd(2,2,N,1); 5+5*betarnd(3,2,N,1); 6+betarnd(2,2,N,1)];
```

Define theoretical pdf for sum of distributions (eg. mixture of beta distributions with different parameters in case 3) - use `normpdf`, `unifpdf` and `betapdf` in case 1, 2 and 3 respectively. Note that the integral of the new pdf must be equal to one. In each case, plot data points from sample (marked with $+$) and pdf (solid line) at one figure. At the plot mark the theoretical mean value of component distributions - see table:

No	distribution	p.d.f.	mean
1	normal $\mathcal{N}(\mu,\sigma^2)$	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	μ
2	uniform $\mathcal{U}[a,b]$	$f(x)=\frac{1}{b-a}\mathbf{1}\{a\leqslant x\leqslant b\}$	$\mu=\frac{1}{2}(a+b)$
3	$Beta(\alpha,\beta)$	$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}\mathbf{1}\{0\leqslant x\leqslant 1\}$	$\mu=\frac{\alpha}{\alpha+\beta}$

Task 2. */1 points/* Try to fit pdf with different bandwidths Δ from 0.1 to 2.0 - see `ksdensity` in MATLAB. Create a figure with several kernel estimators of pdf on it and the theoretical pdf (for all three cases separately).

Task 3. */2 points/* For every bandwidth Δ from 0.1 to 2.0 calculate MSE (mean square error) between a theoretical joint distribution and a curve fitted by kernels. Plot the MSE in function of bandwidth. Repeat it for all cases in task 1. Is the default bandwidth the best?

Task 4. */1 points/* Try different types of kernel: normal, box, triangle and Epanechnikov and observe how the result of fitting changes.

/Total: 5 points/